ON THE ESTIMATION OF LONG WAVE PARAMETERS FROM A SINGLE ADCP MOORING.

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INTRODUCTION. This paper is focused on the estimation of wave length and periods of coastal trapped waves (Talpsepp, 2006, Lilover, Talpsepp, 2014) from single ADCP moorings. In case of single mooring it is possible to estimate the wave period but not the wave lenght (or phase speed). If the assumption that waves are propagating along isobaths is valid we need at least two measuring points at the same time to estimate the wave length. Using bottom mounted ADCP we try to exclude this requirement and to estimate the wave length using current measurements at one single point in case of vertically inhomogeneous mean flow.

CLASSICAL DOPPLER EFFECT. In classical physics, where the speeds of source and the receiver relative to the medium are lower than the velocity of waves in the medium, the relationship between observed frequency f and emitted frequency f_0 is given by

$$f = \left(\frac{c + v_r}{c + v_s}\right) f_0$$

where
C is the velocity of waves in the medium;
V_r is the velocity of the receiver relative to the medium; positive if the receiver is moving towards the source (and negative in the other direction);
V_s is the velocity of the source relative to the medium; positive if the source is moving away from the receiver (and negative in the other direction)

Doppler effect of water flow around a swan

MEASUREMENTS. The data used were obtained by a bottom-mounted ADCP deployed near the southern coast of the Gulf of Finland at the position 59N 27.400 24E 09.96 about 6 km offshore in the region of the coastal slope. The instrument was deployed for the period between 13 March and 30 June 2009. The mooring depth was 50 m and currents data at 20 different levels were used to in the Gulf of Finland were analyzed during 105 days.

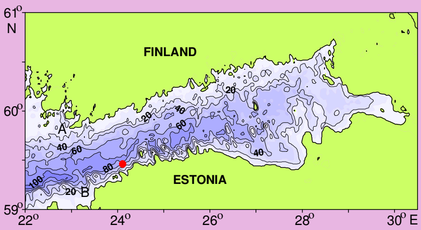

Fig. 1. THE LOCATION OF ADCP MOORING IN THE GULF OF FINLAND IN 2009

METHOD. Topographically trapped waves can be presented as

$$U = P(x,z) \exp(2\pi i (ly - ft)), \tag{1}$$

where x is directed cross isobath, **y** is directed along isobathes, **z** is directed from the surface downwards, $\mathbf{f_o}$ is the wave frequency. We denote the actual phase speed by **f** (independent on vertical coordinate **z**), mean velocity by $\mathbf{V_m}$ (generally $\mathbf{V_m = V_m(z)}$), where the mean velocity is the avarage velocity over many wave periods, wave period we denote by **T** (actual period) or $\mathbf{T_o}$ (observed period). Let **L** be the wave length

$$L = P * T,$$

where **P** and **T** are actual phase speed and actual period, correspondingly. If we observe the frequency of current velocity variabilty in the moving medium, then the frequency will increase when the medium (mean velocity) is directed to the same direction as the phase of current velocity and decrase when the medium is moving to the opposite direction. Using $\mathbf{f = 2\pi/T}$ or $\mathbf{f_o = 2\pi/T_o}$ we will get the basic system of N equations for two unknown parameters **P** and **T**

$$1/T_o(z) = (P + V_m(z))/(P\,T). \tag{2}$$

Formula (2) is valid for **z= z1,..., zN**, where **z1,...,zN** are different depths, observed period $\mathbf{T_o(z)}$ and mean speed $\mathbf{V_m(z)}$ are found from measurements. Thus, we can solve equations for actual period **T** and phase speed of the wave **P** as follows

$$T_o = PT/(P + V_m(z))\,|\, z = z1, z2, \ldots$$

or

$$T = T_o (P + V_m(z))/P\,|\, z = z1, z2, \ldots$$

and

$$T_o(z1)(P + V_m(z1))/P = T_o(z2)(P + V_m(z2))/P$$
$$T_o(z1)(P + V_m(z1)) = T_o(z2)(P + V_m(z2))$$
$$(P + V_m(z1)) = (P + V_m(z2))$$

From here we can find

$$P = (T_o(z2) V_m(z2) - (T_o(z1) V_m(z1)) / (T_o(z1) - T_o(z2)) \tag{3}$$
$$T = T_o(z1)(P + V_m(z1))/P \tag{4}$$
$$L = P*T, \tag{5}$$

where **T** and **P** in (5) are found from (3) and (4).

SPECTRAL CALCULATIONS. In Fig.2 the energy spectra of the velocity u component at different water levels is presented. The spectra were computed for the time period from 13 March up to 15 of April 2009 using the Welch overlapping method. The main peak in all spectra was at 4.5–5.5 days. The spectra are presented in coordinates *ω·S(ω)*, where *ω* is frequency and *S(ω)* energy spectrum. In that presentation the energy in the corresponding frequency interval is proportional to the area under the curve and the energy in different frequency intervals and different levels can be compared. In Fig.2 the spectra for 7 water levels are presented. Most spectra have one maximum at the periods of 4.5–5.5 days. But in Fig. 2 we notice that in the energy maximums (peaks of the spectra) have a shift that according (1) is not expected.

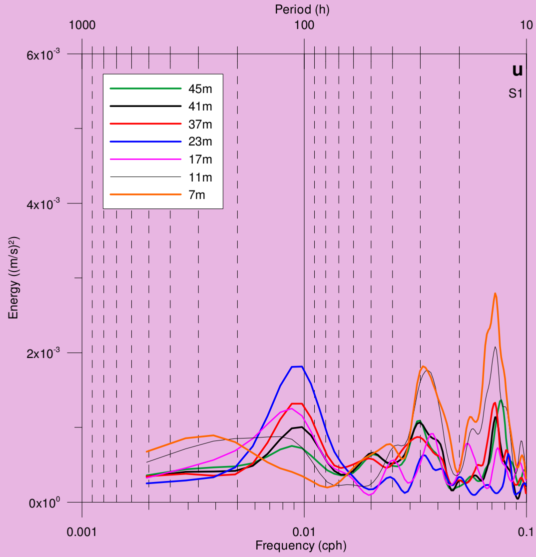

Fig.2. The energy spectra of the current component u at different depths in the Gulf of Finland near Suurupi peninsula in March-April 2009 (adopted from (Talpsepp, Lilover, 2011)).

DISCUSSION: The calculated spectra show that there is much more energy below the surface layer in the low-frequency range. The maximum of the kinetic energy was observed at the level of 23 – 27 m from the sea surface. Observed variability was absent in the surface layer, thus, direct influence of the wind would be excluded. One possible mechanism for the generation of intensive low-frequency 4.5 to 5.5-day current oscillations may be topographic waves ias a response to the wind stress on the surface layer. Topographically (coastal and bottom-) trapped waves are often observed over sloping bottom (see Huthnance [1], for example), having the maximum kinetic energy in the region of bottom slope.

As it was described above, the calculated spectra in Fig.2 show that the peaks in the spectra for different levels are shifted in relation each other. As the peak in spectra corresponds to the observed period we can conclude that these observed periods are slightly different at defferent water levels. Near the bottom the periods of oscillations are shorter (green line has the maximum near frequency 0.009 1/s but violet line approximately at 0.008) and in the medium layer periods are longer. As the theory of topographic waves expects the vertically constant wave parameters then the observed periods may be influenced by vertically inhomogeneous mean flow changeing the real periods at different levels differently. Looking at the vertical distribution of the mean flow (mean over the sub-period I) we notice that it is stronger in the upper layer and less intensive in the lower layer being in range 1-5 cm/s.

Using formulae (3) and (4) for levels 17 m and 41 meters (that means N=2) we can calculate T and P and using (2) also the wave length. Actually, it is possible to combine any two levels in calculations. Slightly different values for T, P and L can be optimized in the way that the error (difference from mean value) will be minimal. In present case we got the value of wave length of 15 km, with corresponds to the internal Rossby radius estimation and to the estimation from model of topographic waves for that region and stratification.

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